

## Comparison of Boost Phase Prediction Methods For Missile Defense

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**This paper addresses the issue of predicting where a boosting threat missile will be several minutes in the future. The prediction issue is important for missile defense because it can be a key factor in determining how heavy an exoatmospheric interceptor has to be. Simplified examples are presented comparing the effectiveness of two prediction methods in terms of accuracy and robustness. Finally the prediction methods are evaluated in an end-to-end engagement simulation to see how they influence the lateral divert requirements of a kill vehicle.**

### Introduction

Intercepting ballistic missiles in their boost-phase has long been viewed as an attractive option because ballistic missiles are easy to detect and track in their boost phase and, if the intercept is successful; the entire missile payload is destroyed or diverted in a single shot, thus thinning the raid size for subsequent missile defense layers. In addition, decoys and other countermeasures generally are more difficult to devise for boosting missiles compared to warheads in their midcourse phase of flight. However, the primary difficulty for boost-phase ballistic missile defense (BMD) is the need to be close enough to the target to destroy it in the short amount of time available (60-300 seconds, depending on the ballistic missile's range, minus the time required to detect the missile launch and track the booster with sufficient accuracy to engage it).

Airborne platforms have distinct advantages for boost-phase defense (as well as some

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liabilities). Being mobile, airborne boost-phase interceptors can be deployed rapidly and

flexibly to a theater of conflict and their deployment can be scaled to match the threat. To some extent, the limited carrying capacity of airborne platforms is compensated for by the fact that airborne interceptors can be smaller than surface-based interceptors because

they experience less drag during fly out and they may get an initial boost from the aircraft's speed at the time of interceptor launch.

In order to fire an interceptor at a boosting target we must first make a prediction of where the target will be in the future. Generally we desire to lead the target so that if the interceptor is at a velocity disadvantage it will still be kinematically possible to reach the intercept point. The amount of error there is in the predicted intercept point or PIP is known as the PIP error. It is believed that the larger the PIP error, the more divert fuel will be required by the kill vehicle portion of the interceptor to take out the error and hit the target. Large divert fuel requirements for the kill vehicle will result in a heavier kill vehicle which in turn will result in a heavier and more expensive interceptor. The size of the PIP error can be one of the major considerations in determining the weight of the kill vehicle. Thus from a system point of view, it appears that methods for significantly reducing the PIP error can be very important. In this paper two possible methods for

determining the PIP will be evaluated and compared against three different types of boosting targets in terms of the resultant PIP error. The three targets considered are a one-stage liquid propellant intermediate range ballistic missile (IRBM), a two-stage liquid propellant intercontinental ballistic missile (ICBM) and a three-stage solid propellant ICBM.

Design is an iterative process. The comparisons done in this paper are performed without considering the effects of measurement noise on the prediction errors. This assumption is made to make it easier to understand the advantages of one prediction method over another. Future iterations in the design process must consider the influence of the measurement noise on the state estimates.

## Possible Prediction Methods

A physics based method of prediction for a boosting target can be based on a constant thrust model. With a constant thrust and time varying weight (due to propellant consumption during the boost phase) we can find the derivative of the boosting target's thrust to weight ratio as

$$\frac{d}{dt}\left(\frac{T}{W}\right) = \frac{W\dot{T} - T\dot{W}}{W^2} = -\frac{T\dot{W}}{W^2}$$

where  $T$  is the thrust and  $W$  is the weight in the English system of units. Recall that the specific impulse of the rocket's fuel  $I_{SP}$ , in units of seconds, is defined as

$$I_{SP} = -\frac{T}{\dot{W}}$$

where  $\dot{W}$  is the rate of change of the weight. Substitution of the preceding expression into the equation for the derivative of thrust to weight ratio yields a version of the rocket equation or

$$\frac{d}{dt}\left(\frac{T}{W}\right) = \frac{\left(\frac{T}{W}\right)^2}{I_{SP}}$$

Since thrust divided by weight is the boosting target's acceleration  $a_T$  in units of  $g$ , we can say that

$$\dot{a}_T = \frac{a_T^2}{gI_{SP}}$$

The preceding differential equation is integrated forward from the current time to the desired intercept time at each guidance update. In addition, to finding the acceleration magnitude of the target, it is also required to assume a direction of the booster's acceleration vector. A common assumption is that the target always flies a gravity turn<sup>1</sup>. Then at each instant of time the differential equations for the acceleration components of the booster in an Earth-centered coordinate system are given by

$$a_x = \frac{-gm}{(x^2 + y^2)^{1.5}} + a_T \frac{\dot{x}}{(\dot{x}^2 + \dot{y}^2)^{.5}}$$

$$a_y = \frac{-gm}{(x^2 + y^2)^{1.5}} + a_T \frac{\dot{y}}{(\dot{x}^2 + \dot{y}^2)^{.5}}$$

where  $gm$  is the universal gravitational constant. The first term in the preceding differential equations is due to gravity and the second term is the gravity turn portion of the boosting target's acceleration. The preceding differential equations are integrated twice to get the future location ( $x_F$ ,  $y_F$ ) of the target and to monitor the acceleration magnitude  $a_T$ . If the acceleration magnitude exceeds the assumed maximum acceleration limit of the booster  $a_{MAX}$ , then the differential equations are reinitialized with  $a_T$  being set to minimum acceleration capability of the booster  $a_{MIN}$ . If time exceeds the estimated burn time of the booster we assume that the booster is in it's ballistic phase of flight where  $a_T=0$ . Thus to implement the gravity turn prediction method we need to have *a priori* estimates of the booster's fuel specific impulse, the minimum and maximum acceleration capabilities of the booster and the booster burnout time.

Another way of predicting the future location of a boosting target is to use a simple three-term Taylor series expansion based on the current position, velocity and acceleration of the target. Using this method, the future location of the target ( $x_F$ ,  $y_F$ ) is given by

$$x_F = x + \dot{x}t_{go} + 0.5\ddot{x}t_{go}^2$$

$$y_F = y + \dot{y}t_{go} + 0.5\ddot{y}t_{go}^2$$

where  $t_{go}$  is the time to go from the desired intercept time to the current time. The Taylor series method makes no *a priori* assumptions concerning the booster characteristics or it's method of guidance.

## How Long-Range Boosters Fly

In order for a booster to hit it's intended target it must have a feedback method of control, as do all guided missiles. Long-range liquid fueled rockets, where the booster portion can thrust terminate, usually employ some form of Lambert guidance while solid fueled rockets that do not have a thrust termination system and must consume all of their fuel, usually employ some form of General Energy Management (GEM) guidance. It is common practice for the these solid propellant long-range rockets to fly straight up for a period of time in order to get out of the atmosphere as quickly as possible and then, while still in the atmosphere, perform a gravity turn type of maneuver in order to reduce the resultant drag and loading effects. Only after the

dynamic pressure reduces to a certain level (i.e., booster design dependent and different for each type of rocket) does Lambert or GEM guidance begin. In this paper the gravity turn and Taylor series methods of prediction will be compared for a one-stage IRBM, a two-stage liquid-fueled ICBM, and a three-stage solid-fueled ICBM.

Both Lambert and GEM guidance involve the numerical solution to Lambert's problem<sup>3,4</sup>. Essentially, at each instant of time if you know where you are and where you want to go and how long it should take you to get to your destination, the solution to Lambert's problem tells you the magnitude and direction of the required velocity vector. If, in two dimensions, the solution to Lambert's problem yields a velocity solution  $V_{LAMx}$  and  $V_{LAMY}$  we compute a velocity to be gained which is the difference between the Lambert solution and our instantaneous velocity ( $V_x, V_y$ ) or

$$\Delta V_x = V_{LAMx} - V_x$$

$$\Delta V_y = V_{LAMY} - V_y$$

The total velocity to be gained  $\Delta V$  can be found from

$$\Delta V = \sqrt{\Delta V_x^2 + \Delta V_y^2}$$

With Lambert guidance we attempt to align the booster's thrust vector with the velocity to be gained vector yielding acceleration components as

$$a_x = \frac{-gm}{(x^2 + y^2)^{1.5}} \frac{x}{\Delta V} + a_T \frac{\Delta V_x}{\Delta V}$$

$$a_y = \frac{-gm}{(x^2 + y^2)^{1.5}} \frac{y}{\Delta V} + a_T \frac{\Delta V_y}{\Delta V}$$

With GEM guidance all of the fuel must be consumed and so excess energy is wasted in a circular arc. Details of both Lambert and GEM guidance can be found in Ref. 5.

## One-Stage IRBM

Figure 1 displays the longitudinal acceleration of a generic one-stage IRBM with a burn time of 168 s. The IRBM is capable of traveling more than 3000 km. We can see that the initial IRBM acceleration starts out at approximately 1.5 g and increases parabolically to a maximum of approximately 13 g. The specific impulse of the fuel is 273 s.

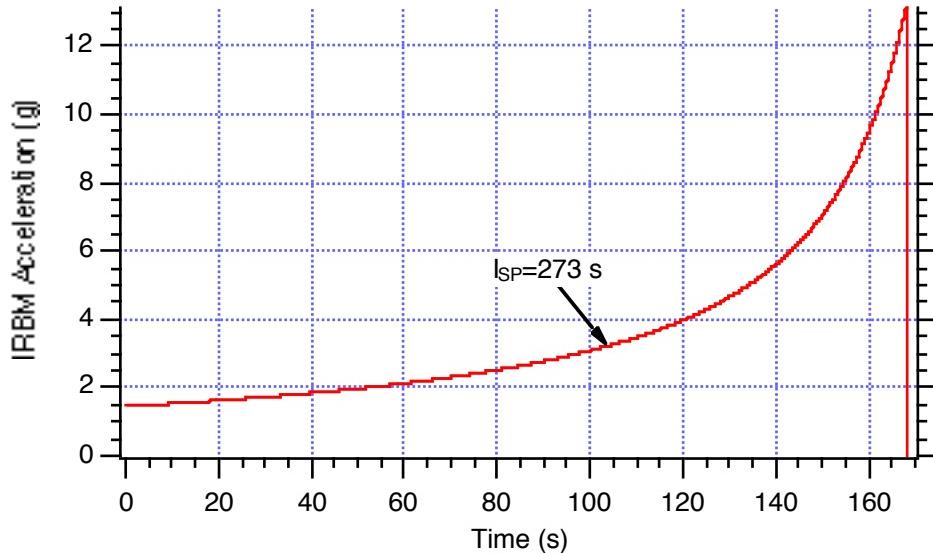


Figure 1 One-stage IRBM acceleration

For academic purposes let us see what happens if the IRBM flies a gravity turn all the way through its boost phase. In this case, where the IRBM lands depends on its initial kick angle (angle between horizontal and velocity vector) and many other practical considerations. However, for now, we will ignore the other practical considerations and assume that the IRBM goes straight up for 10 s and then performs a gravity turn with an initial kick angle of 89.8 deg. The resultant one-stage IRBM trajectory is shown in Fig. 2. We can see that for the chosen kick angle the IRBM travels nearly 900 km with a trajectory apogee of approximately 1300 km. Different initial kick angles will yield vastly different trajectories.

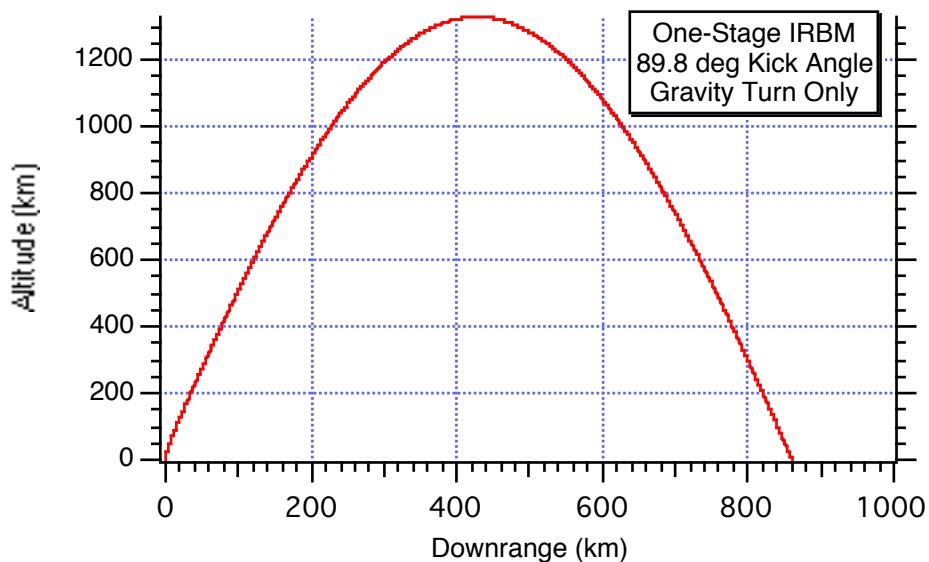


Figure 2 One-stage IRBM Trajectory performing only a gravity turn

Figure 2 presents the boost phase portion of the IRBM trajectory with 10 s time tics. The 170 s time tic is 2 s after the end of the boost phase and the 100 s time is a reference point.

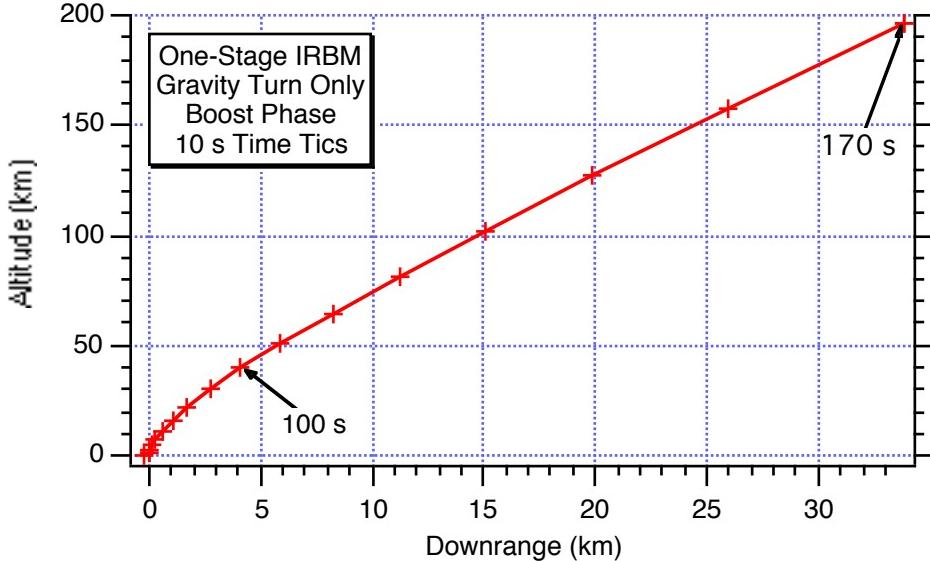


Figure 3 Boost phase portion of one-stage IRBM trajectory performing only a gravity turn

Consider the case in which the single-stage IRBM goes straight up for 10 s and then performs a gravity turn for the remainder of the boost phase. Figure 4 compares both prediction methods in terms of the PIP error for a case in which we are predicting where the booster will be in 160 s. We can see from Fig. 4 that the Taylor series method yields approximately 100 km of prediction error at the beginning of flight. The Taylor series prediction error decreases as time increases (or  $t_{go}$  decreases). On the other hand, the gravity turn prediction method yields zero PIP error after 10 s (after target goes straight up). In this case the gravity turn prediction method is perfect because the target is actually performing a gravity turn and it is assumed that the fuel specific impulse and target burnout times are known precisely.

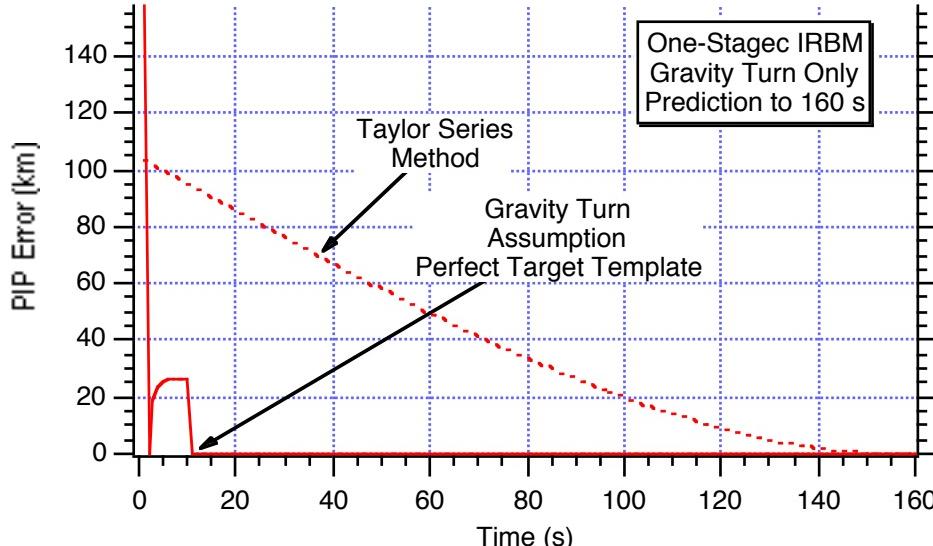


Figure 4 Gravity turn assumption yields smaller PIP errors if one-stage IRBM is actually performing a gravity turn all during boost phase

The dynamic pressure  $Q$ , in  $\text{lb}/\text{ft}^2$ , is given by

$$Q = 0.5\rho V^2$$

where  $V$  is the booster's velocity in  $\text{ft}/\text{s}$  and  $\rho$  is air density in  $\text{slug}/\text{ft}^3$ . In the English system of units air density  $\rho$  can be approximated by

$$\rho = 0.0034e^{-alt/2200}$$

where  $alt$  is the booster altitude in ft.

The dynamic pressure as a function of altitude for the IRBM performing a gravity turn appears in Fig. 4. If we assume that IRBM guidance cannot begin until the dynamic pressure drops below  $600 \text{ lb}/\text{ft}^2$  (i.e., actual number depends on booster design) then we can say that for this example closed-loop guidance can not begin until 65 s after the IRBM takes off.

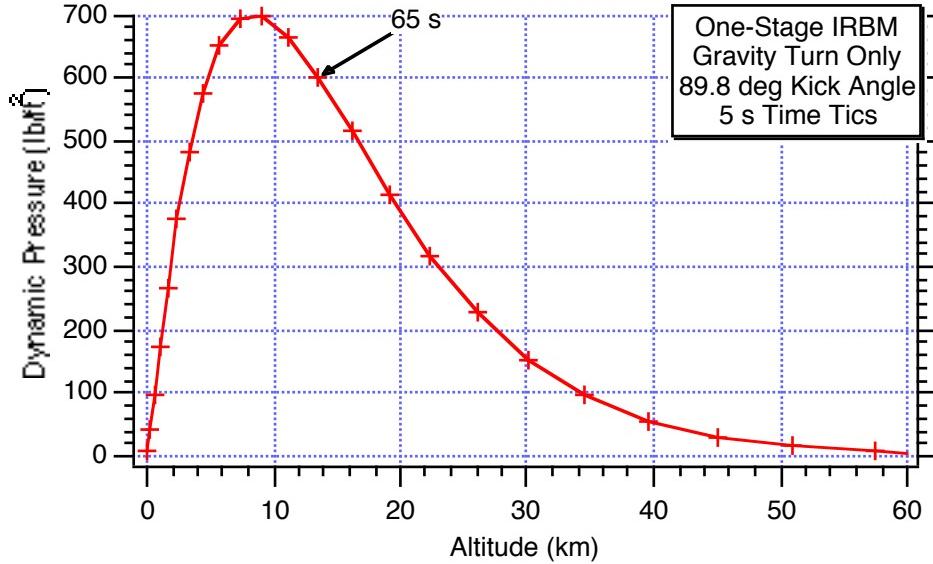


Figure 5 Dynamic pressure drops below 600 lb/ft<sup>2</sup> at 14 km altitude or 65 s after IRBM takes off

Another case was run in which the IRBM went straight up for 10 s, performed a gravity turn until 65 s after launch and then switched to Lambert guidance so that it would hit an impact point 2000 km downrange. The entire IRBM trajectory is depicted in Fig. 6 and the boost phase portion of the trajectory, with 10 s time tics, appears in Fig. 7. We can see that the boost phase portion of the trajectory differs considerably from the all gravity turn boost phase trajectory of Fig. 3.

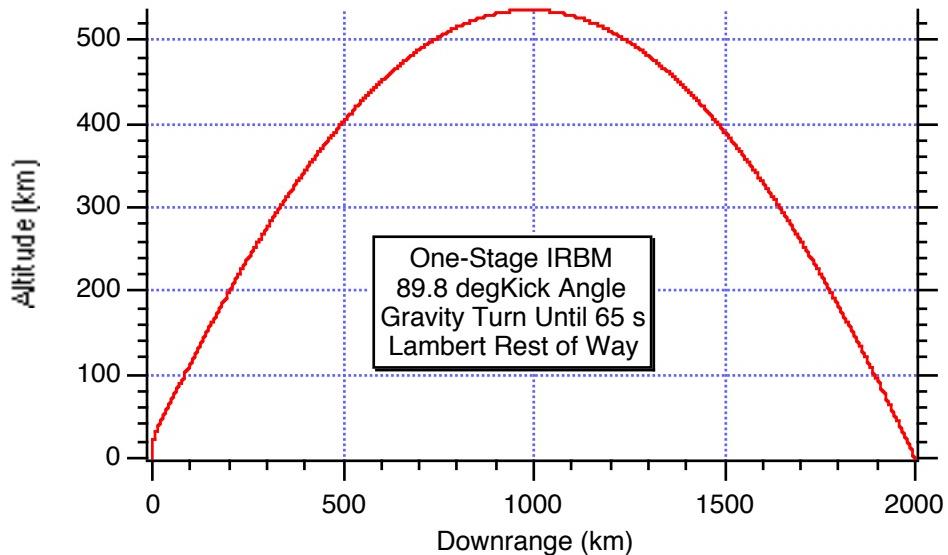


Figure 6 One-stage IRBM trajectory due to performing a gravity turn and then Lambert guidance

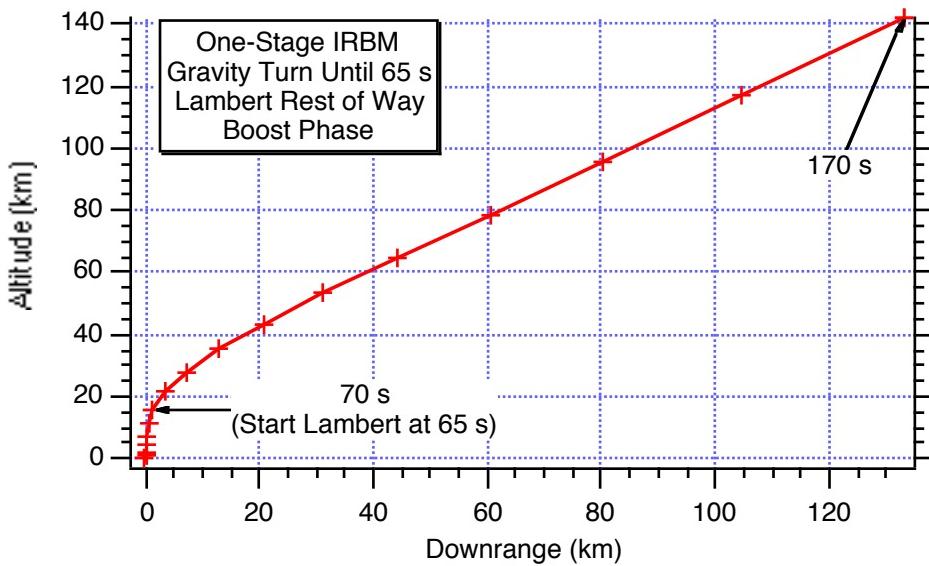


Figure 7 Boost phase portion of one-stage IRBM trajectory performing a gravity turn and then Lambert guidance

The PIP errors for the Taylor series and gravity turn methods of prediction are compared in Fig. 8. As was mentioned before, the actual specific impulse of the fuel is 273 s. For the gravity turn method of prediction we assume that we lack knowledge of the fuel specific impulse and consider using 200 s and 300 s (i.e., practical bounds) for our estimates. Here we can see that the gravity turn method yield smaller PIP errors for the first 65 s when the IRBM is actually flying a gravity turn. For the next 10 s the Taylor series method yields smaller PIP errors (when the IRBM is flying Lambert guidance) and then for the remainder of the flight the gravity turn method yields smaller PIP errors. After 65 s, the gravity turn method of prediction does not appear to be sensitive to the estimate of the fuel specific impulse. In the future we shall always assume the estimated fuel specific impulse to be 300 s because that seems to yield the smallest PIP errors.

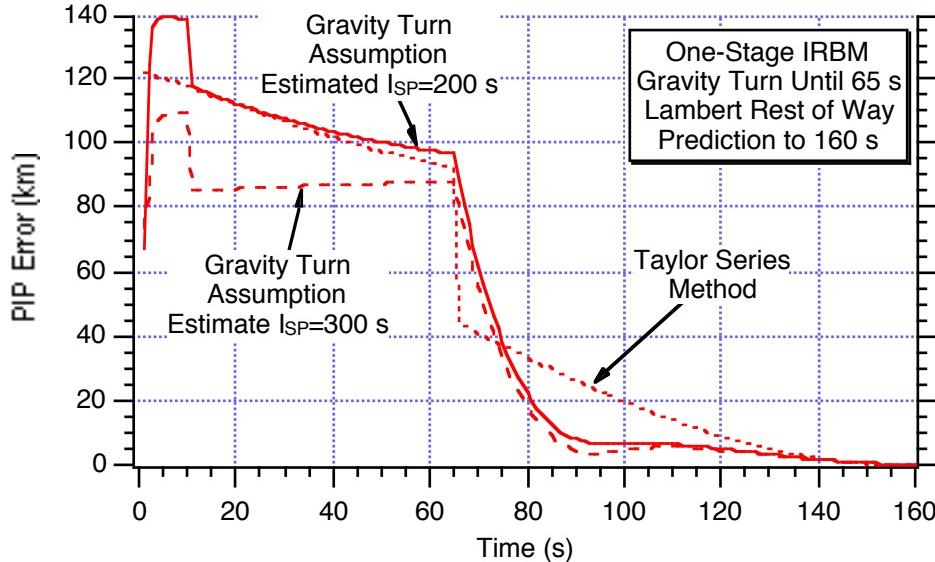


Figure 8 It is better to over estimate fuel specific impulse if gravity turn prediction method is used

In the preceding figure it was assumed that the exact burnout time (168 s) of the IRBM was known in advance. Figure 9 shows that if we assume that the IRBM burnout time is 180 s (rather than 168 s) and we assume that the fuel specific impulse is 300 s (rather than 273 s), the gravity turn method still yields smaller PIP errors than the Taylor series method. Figure 10 shows that if we assume that the IRBM burnout time is 158 s (rather than 168 s) and we assume that the fuel specific impulse is 300 s (rather than 273 s), the gravity turn method still yields smaller PIP errors than the Taylor series method.

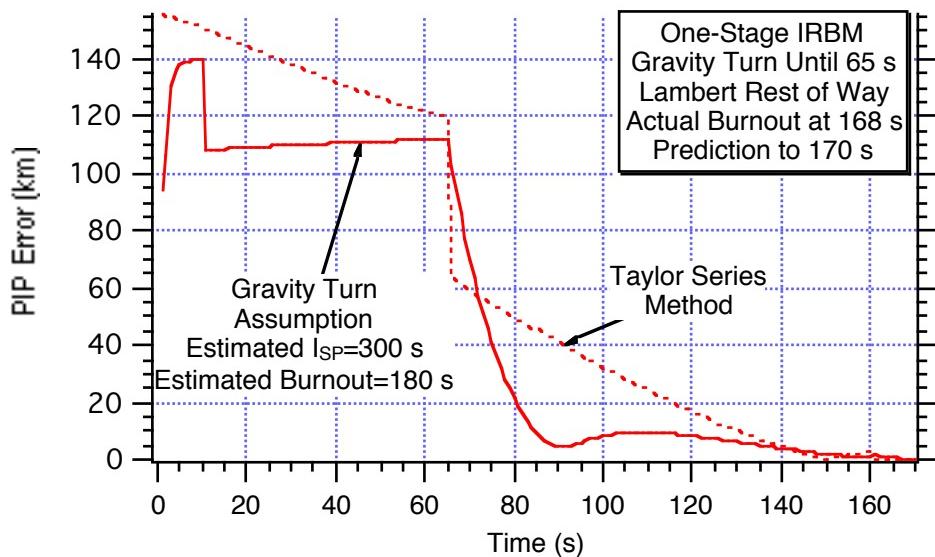


Figure 9 Gravity turn method yields smaller PIP errors than Taylor series method if burnout time is overestimated

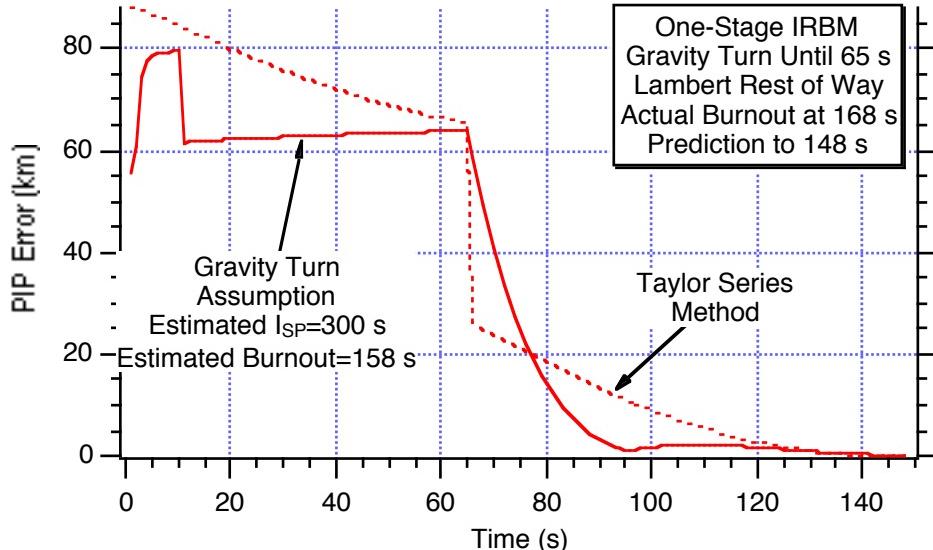


Figure 10 Gravity turn method yields smaller PIP errors than Taylor series method if burnout time is underestimated

Another case was run in which it was attempted to see how robust each prediction method was for intentional changes in the target trajectory. Figure 11 depicts complete trajectories for the nominal case in which the IRBM traveled 2000 km and another case in which the IRBM initially guides on the 2000 km impact point but at 100 s, changes its mind, and heads for an impact point 1000 km downrange.

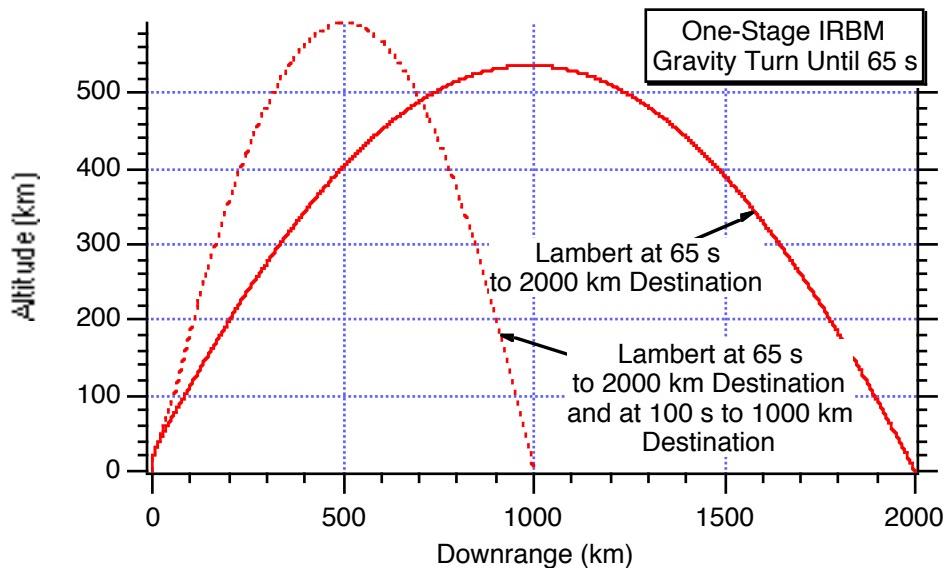


Figure 11 Example of IRBM range-change trajectory starting at 100 s

We can see from Fig. 12 the gravity turn method yields smaller PIP errors than the Taylor series method for the first 100 s. However, at 100 s, when the destination of the IRBM changes, the Taylor series method yields the smallest PIP errors.

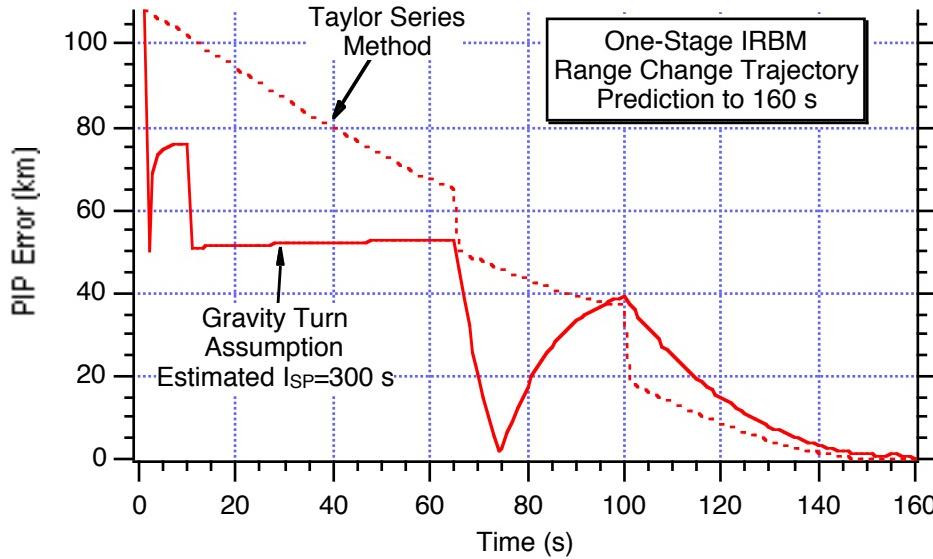


Figure 12 Comparison of prediction errors for range-change trajectory

In this section we have seen that for the single-stage IRBM case, the gravity turn prediction method generally yields smaller PIP errors than the Taylor series method. We have also seen that the gravity turn method is fairly insensitive to lack of knowledge of the IRBM's specific impulse and burnout time. However a range-changing trajectory appears to favor the Taylor series method of prediction. Let us now see what happens if the booster has multiple stages.

## Two-Stage ICBM

The acceleration profile of a generic two-stage, liquid-fueled ICBM, taken from the American Physical Society (APS) report<sup>6</sup> on boost phase intercept and appears in Fig. 13. Here we can see that the first staging event ends at 120 s and reaches a peak acceleration of 6 g while second stage burnout occurs at 240 s with a peak acceleration of more than 12 g. We can see from Fig. 13 that the fuel specific impulses are slightly different for each stage.

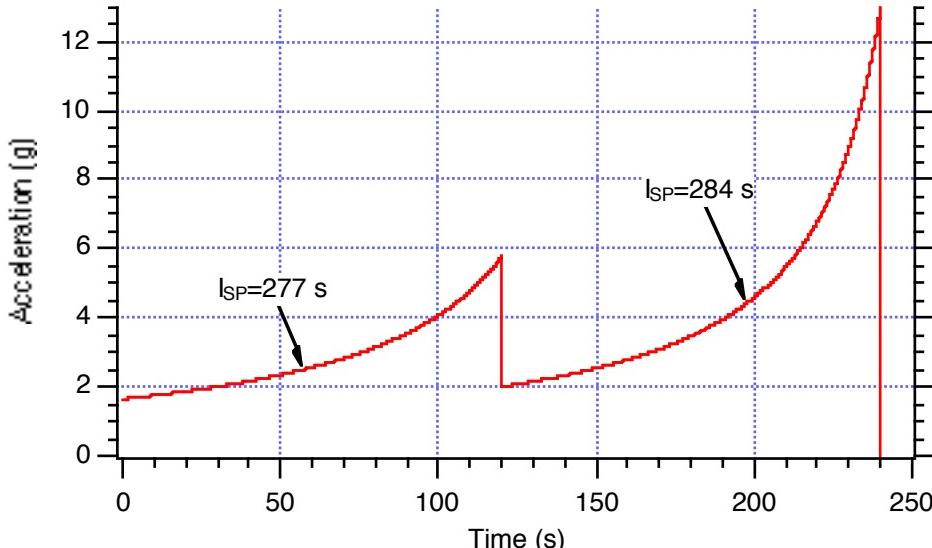


Figure 13 Two-stage ICBM acceleration

Let us pretend that the ICBM goes straight up for 20 s and then flies a gravity turn for the entire boost phase with an initial kick angle of 85 deg. We can see from Fig. 14 that the ICBM travels more than 6000 km with a trajectory apogee of approximately 3500 km.

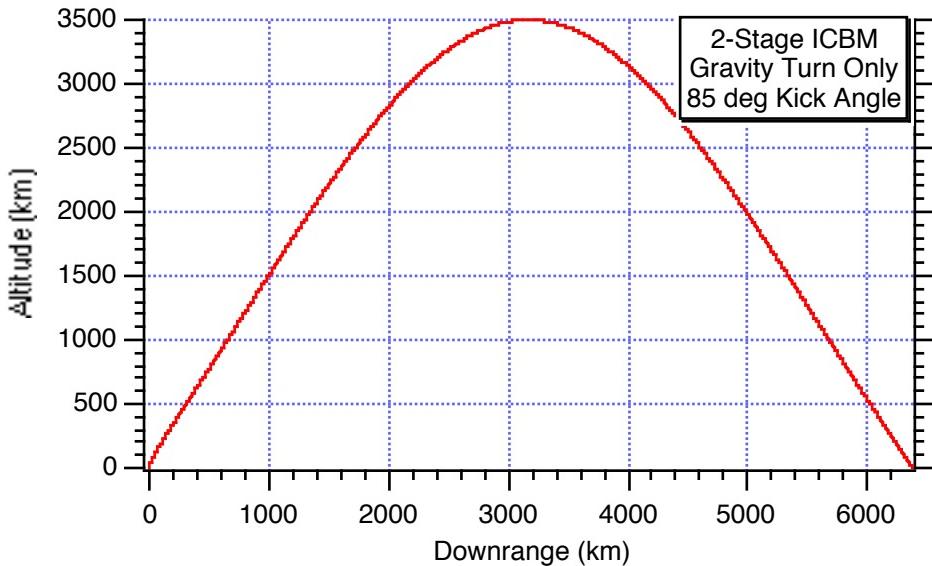


Figure 14 Two-stage ICBM Trajectory performing only a gravity turn

Figure 15 presents the boost phase portion of the two-stage ICBM trajectory with 10 s time tics. The 240 s time tic is the end of the boost phase and the 100 s time tic is for reference purposes.

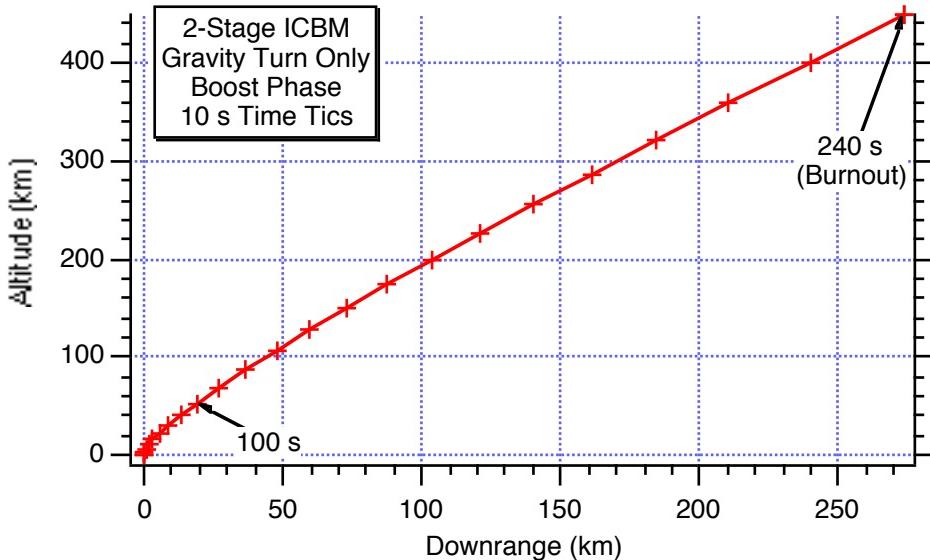


Figure 15 Boost phase portion of 2-stage ICBM Trajectory performing only a gravity turn

The dynamic pressure as a function of altitude for the two-stage ICBM performing a gravity turn appears in Fig. 16. If we assume that ICBM guidance cannot begin until the dynamic pressure drops below  $600 \text{ lb}/\text{ft}^2$  (i.e., actual number depends on booster design) then we can say that for this example closed-loop guidance can not begin until 65 s after the two-stage ICBM takes off.

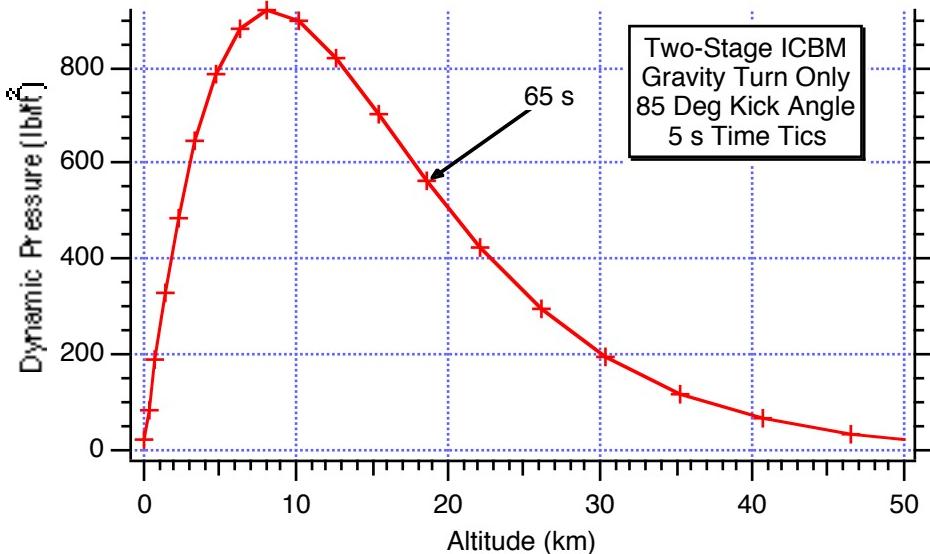


Figure 16 Dynamic pressure drops below  $600 \text{ lb}/\text{ft}^2$  at 19 km altitude or 65 s after two-stage ICBM takes off

Another case was run in which the two-stage ICBM went straight up for 20 s, performed a gravity turn with an initial kick angle of 85 deg until 65 s after launch

and then switched to Lambert guidance so that it would hit an impact point 8000 km downrange. The entire ICBM trajectory is depicted in Fig. 17.

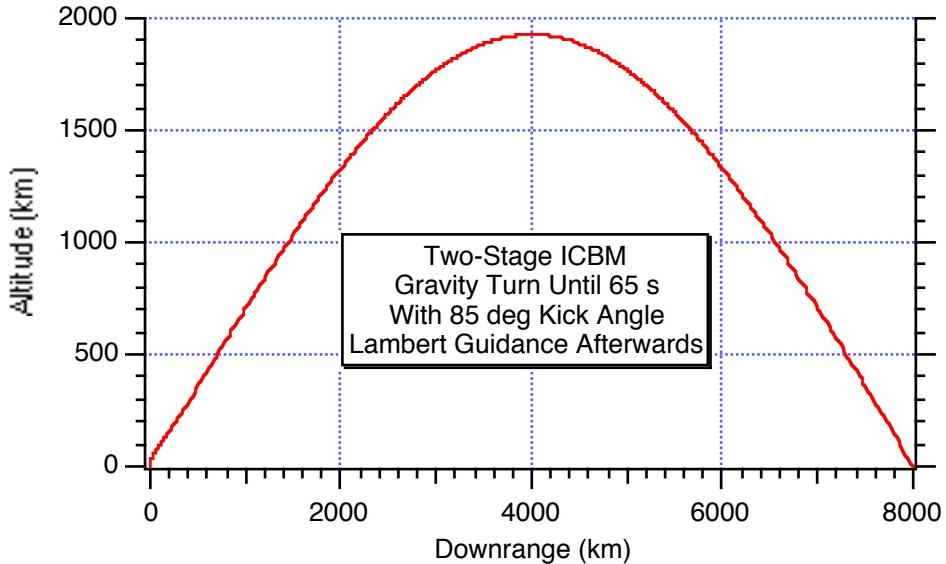


Figure 17 Two-stage ICBM Trajectory performing with gravity turn and Lambert guidance

The Taylor series method of prediction remains the same for the two-stage ICBM but the gravity turn method has to be modified for multiple stage boosters. Recall that the differential equation used to find the booster acceleration magnitude  $a_T$  is given by

$$\dot{a}_T = \frac{a_T^2}{gI_{SP}}$$

where  $I_{SP}$  is the estimated fuel specific impulse. The fragment of code used to limit and reset the magnitude of booster acceleration is given by

```

IF(AT>XNLIM)THEN
    AT=ATIC
    ATOLD=ATIC
ELSEIF(T>TPZ)THEN
    ATP=0.
    ATPOLD=0.
ENDIF

```

In the above logic AT represents the booster acceleration magnitude  $a_T$ , XNLIM represents the maximum expected booster acceleration  $a_{MAX}$ , and ATIC represents the minimum expected booster acceleration  $a_{MIN}$ . The estimated booster burnout time is represented by TPZ. For multiple stage rockets XNLIM is set to 10 g and ATIC is set to 2 g. The estimated burnout time is set to the actual value of 240 s for this example.

The PIP errors for the Taylor series and gravity turn methods of prediction appear in Fig. 18. For the gravity turn method of prediction we assume that we lack knowledge of the fuel specific impulse and, based on the IRBM results of the previous section, assume a fuel specific impulse of 300 s for both stages (i.e. actual values are 277 s for first stage and 284 s for second stage). Here we can see that the gravity turn method yields smaller PIP errors for the first 65 s (when IRBM is actually performing a gravity turn) and then the Taylor series method yields smaller PIP errors until 110 s. After 110 s the PIP errors are slightly smaller for the gravity turn method and after 180 s both methods yield comparable PIP errors. Thus the results of this section indicate that for the two-stage liquid-fueled ICBM both methods of prediction are comparable.

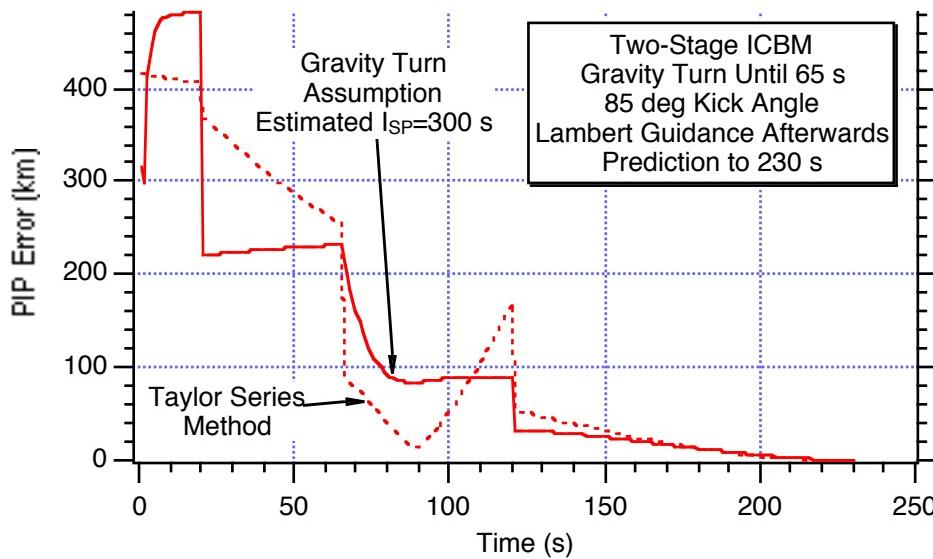


Figure 18 Gravity turn prediction method is slightly better for 2-stage ICBM when burn time is known

### Three-Stage ICBM

The acceleration profile of a generic three-stage, fast burning solid-fueled ICBM, taken from the APS report<sup>6</sup> appears in Fig. 19. Here we can see that the first staging event ends at 65 s and reaches a peak acceleration of 10 g, the second staging event ends at 130 s and reaches a peak acceleration of 10 g, while the third staging event occurs at 170 s with a peak acceleration of more than 6 g. Figure 19 indicates that the fuel specific impulses are slightly different for each stage.

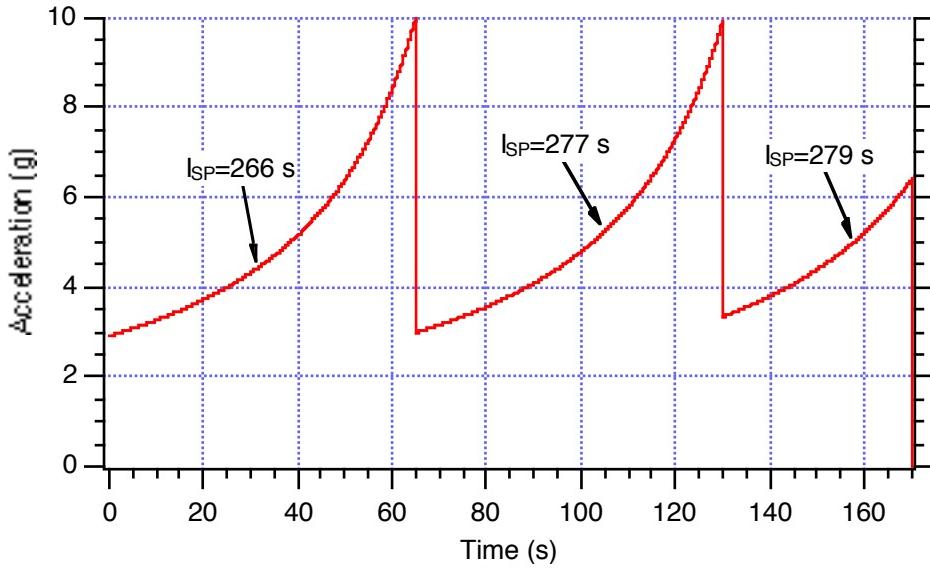


Figure 19 Three-stage ICBM acceleration profile

Let us pretend that the ICBM goes straight up for 20 s and then flies a gravity turn with an initial kick angle of 80 deg. We can see from Fig. 20 that for the 80 deg kick angle the ICBM only travels 3500 km with a trajectory apogee of approximately 4500 km.

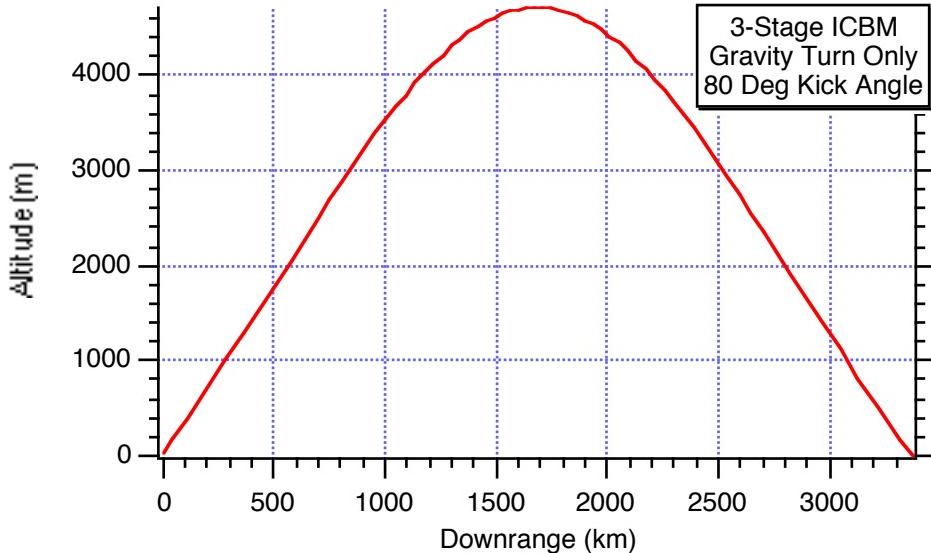


Figure 20 Three-stage ICBM trajectory performing only a gravity turn

Figure 21 presents the boost phase portion of the ICBM trajectory with 10 s time tics. The 170 s time tic is the end of the boost phase and the 50 s time tic represents another reference point.

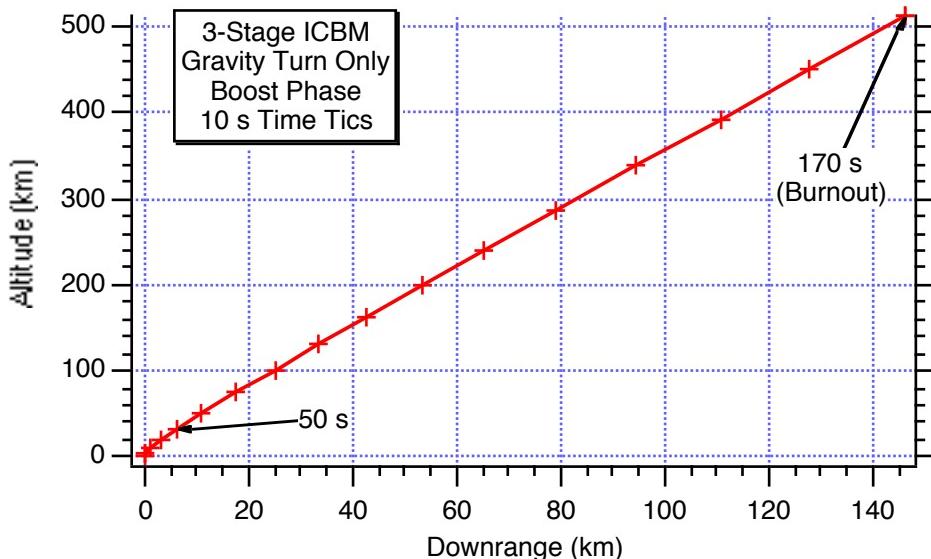


Figure 21 Boost phase portion of 3-stage ICBM Trajectory performing only a gravity turn

The dynamic pressure as a function of altitude for the three-stage ICBM performing a gravity turn appears in Fig. 22. If we assume that ICBM guidance cannot begin until the dynamic pressure drops below  $600 \text{ lb}/\text{ft}^2$  (i.e., actual number depends on booster design) then we can say that for this example closed-loop guidance can not begin until 50 s after the three-stage ICBM takes off.

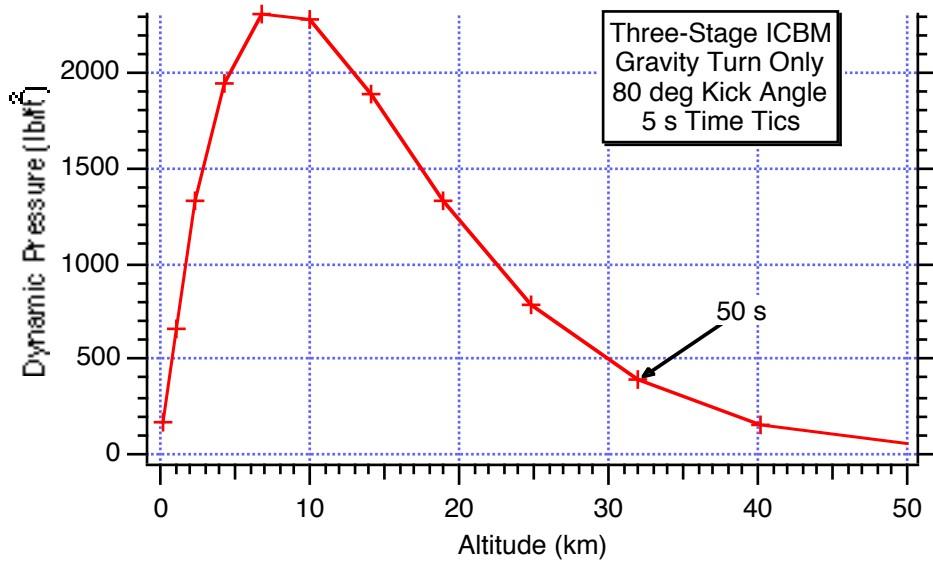


Figure 22 Dynamic pressure drops below 600 lb/ft<sup>2</sup> at 32 km altitude or 50 s after three-stage ICBM takes off

Another case was run in which the three-stage ICBM went straight up for 20 s, performed a gravity turn until 50 s after launch and then switched to GEM guidance so that it would hit an impact point 8000 km downrange. The entire ICBM trajectory is depicted in Fig. 23.

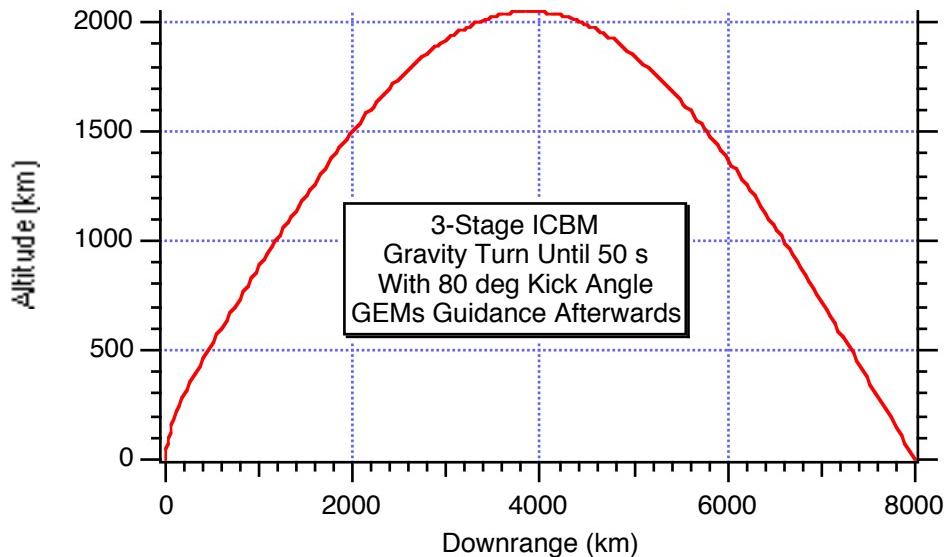


Figure 23 Three-stage ICBM trajectory performing with gravity turn and GEM guidance

The PIP errors for the Taylor series and gravity turn methods of prediction appear in Fig. 24. For the gravity turn method of prediction we assume that we lack knowledge of the fuel specific impulse and, based on previous results, assume a fuel

specific impulse of 300 s for all three stages (i.e. actual values are 266 s for first stage, 277 s for the second stage and 279 s for the third stage). Here we can see that the gravity turn method yield smaller PIP errors for the first 70 s and then the Taylor series method yields smaller PIP errors for most of the remainder of the flight.

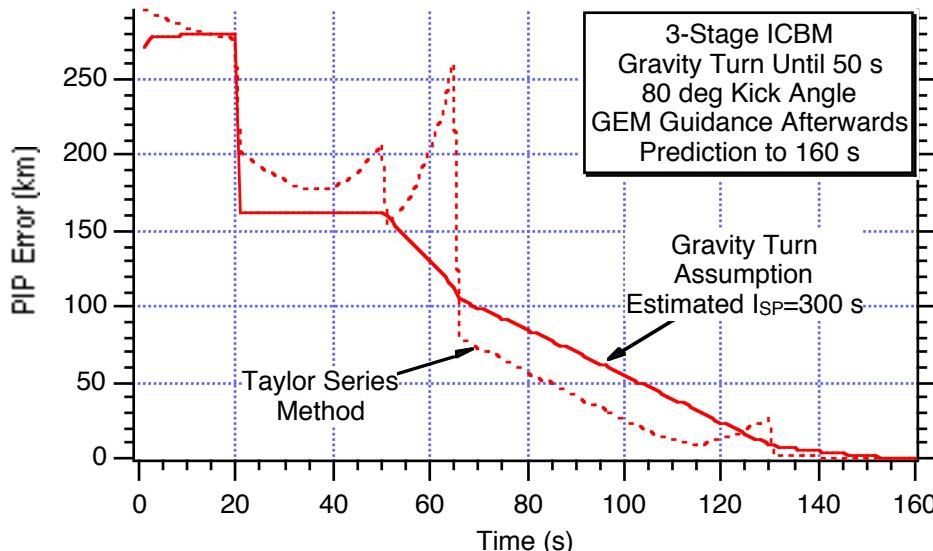


Figure 24 Taylor series prediction method is slightly better for 3-stage ICBM – even when burn time is known

Thus for the three-stage solid-fueled propellant ICBM, where GEM guidance is used, the Taylor series approach to prediction appears to be superior to the gravity turn method

### Engagement Simulation Results

So far we have seen that sometimes the gravity turn method of prediction yields smaller PIP errors and sometimes the Taylor series method of prediction yields smaller PIP errors. So far the results have shown that the superiority of one method over the other depended on the threat and when the prediction was made relative to the threat launch time. In practice, one does not have to choose between the two methods but can develop a hybrid scheme that makes use of the best features of both methods.

To make matters even more confusing it is sometimes possible to have a case in which the PIP errors with one scheme are smaller but the actual divert requirements

of the kill vehicle are larger. To illustrate that the PIP errors are only one of the contributors to the overall kill vehicle divert requirements we need an engagement simulation.

A two-dimensional, end-to-end (i.e., from target launch to intercept) engagement simulation was used to evaluate the effectiveness of the two different PIP calculations in terms of the resultant lateral divert requirements for the kill vehicle. The notional interceptor considered had a burnout velocity of 4 km/s that was reached 20 s after launch from an aircraft platform at 15 km altitude. It is assumed that the notional interceptor guides towards the PIP using Lambert guidance while it is burning and APN guidance is used by the kill vehicle for pursuing the boosting target after the interceptor has burned out. The simulation is deterministic and it is assumed that the current position, velocity and acceleration of the target are known perfectly.

If the only error source in the simulation was PIP error then either the proportional navigation (PN) guidance law or APN guidance law could be used and both would yield identical results. However, since the boosting target appears as a target maneuver to the interceptor, the APN guidance law should do better since APN relaxes the interceptor acceleration requirements due to a maneuvering target. If the interceptor under question could not do APN guidance because of lack of range information than the conclusions reached in this section might change.

Figure 25 presents an engagement geometry in which the notional interceptor is launched at the one-stage IRBM described earlier. The notional interceptor is launched 80 s after the target takes off and the desired intercept time is 160 s. Notice that the interceptor trajectory using the gravity turn prediction method leads the target more than does the Taylor series approach trajectory.

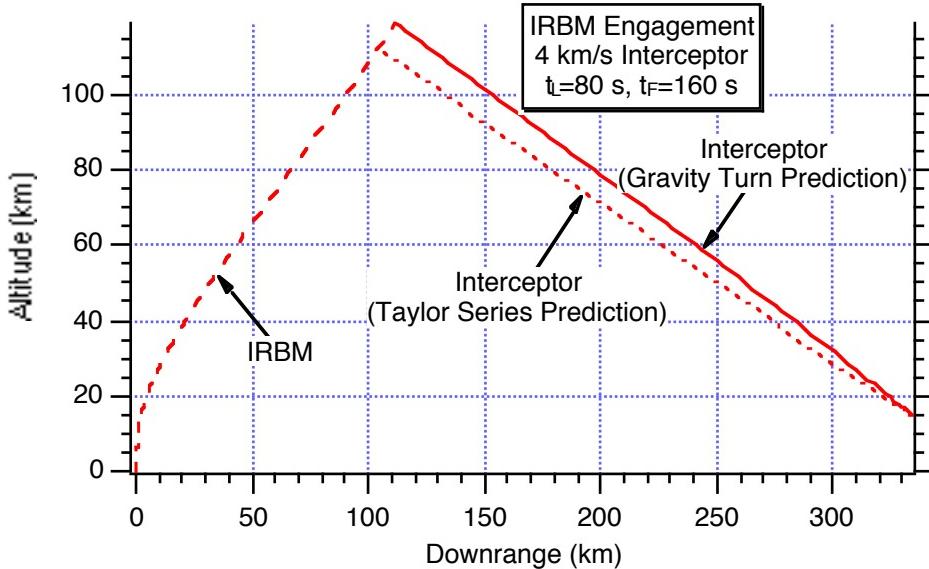


Figure 25 IRBM engagement geometry for interceptors using different PIP algorithms

According to Fig. 8 the PIP error in this IRBM example, using the Taylor series method should be much larger than the PIP error using the gravity turn prediction method. Figure 26 displays the PIP error perpendicular to the line-of-sight (LOS) for both prediction methods. Here we can see that the magnitude of the PIP errors are consistent with those of Fig. 8 between 80 s (when interceptor is launched) to 100 s (when the notional 4 km/s interceptor burns out). We can see that the Taylor series method yields substantially larger PIP errors for the case examined.

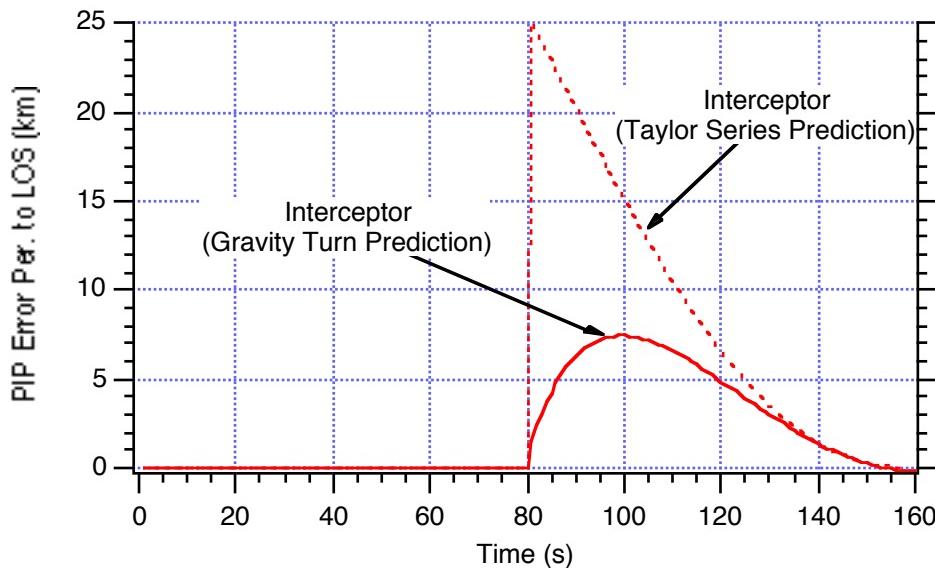


Figure 26 PIP errors are much larger for the Taylor series prediction algorithm

Figure 27 displays the resultant kill vehicle acceleration for taking out the PIP error and chasing the boosting target. If the only source of error in the engagement simulation was PIP error we would expect to see a large acceleration when the kill vehicle initiates APN guidance which would then decrease as the flight progresses. However we can see from Fig. 27 that the acceleration increases as the flight progresses and only decreases near the end of the flight. This type of behavior indicates that the kill vehicle is responding to an apparent target maneuver induced by the accelerating target. As a result of this extra error source, the divert used by the interceptor employing the gravity turn prediction is only slightly less than the one use the Taylor series method. Therefore in this particular engagement, the apparent target maneuver is the biggest contributor to the interceptor's divert requirements. It is important to note that the divert requirements for both engagements appear to be small. When noise and filtering effects (i.e., target states have to be estimated based on sensor measurements) are considered the divert requirements can be several times the size indicated in the figure.

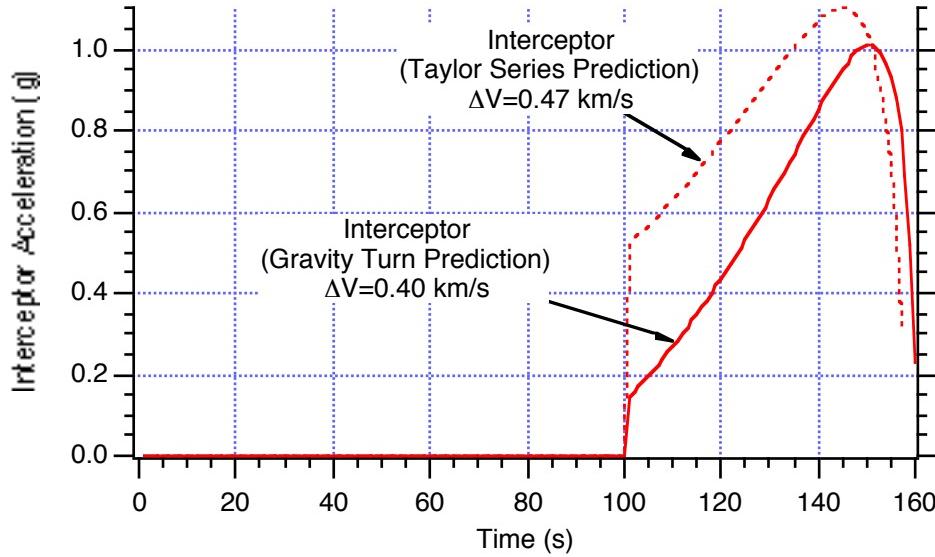


Figure 27 Slightly less acceleration and divert is required when gravity turn PIP method is used

The 2-stage ICBM example was chosen next in order to confirm the importance of the apparent target maneuver. Here an engagement was selected where we launch the notional 4 km/s interceptor with a 20 s burn time 110 s after the target takes off and intercept is desired at 230 s or 10 s before target burnout. Figure 28 displays two interceptor trajectories, each one using a different PIP algorithm. We can see that the

gravity turn assumption again causes the notional interceptor to lead the target more – thus lengthening the flight time.

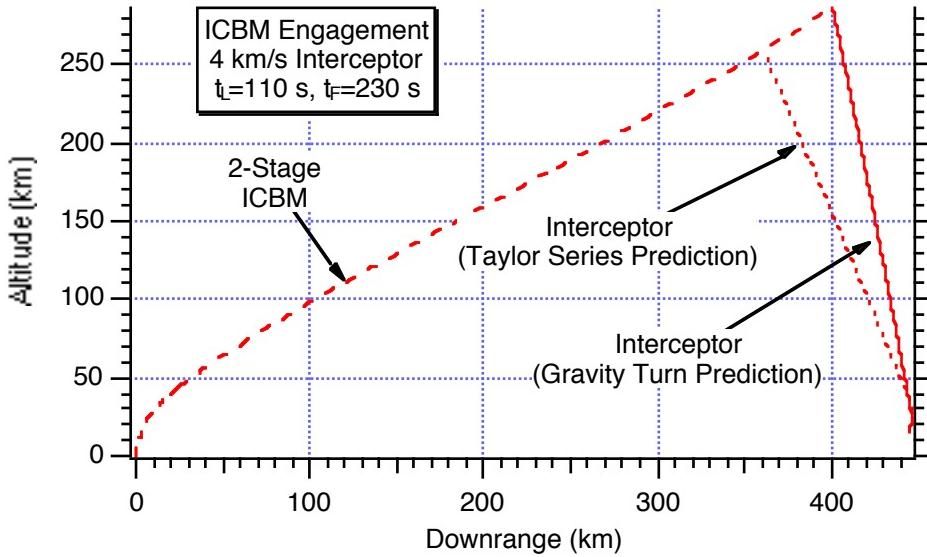


Figure 28 Two-stage ICBM engagement geometry for interceptors using different PIP algorithms

Figure 29 indicates that the Taylor series method yields substantially higher PIP errors than the gravity turn method for the time the interceptor is launched at 110 s and getting up to speed at 130 s while guiding on the PIP. The PIP error numbers for the 110 s to 130 s time frame in Fig. 29 are consistent with those of Fig. 18.

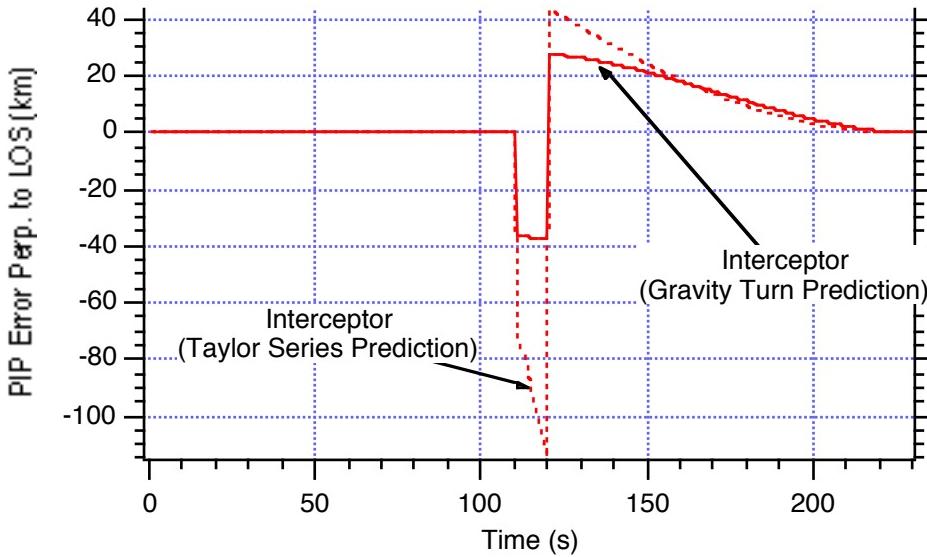


Figure 29 PIP errors are much larger for the Taylor series prediction algorithm

However, Fig. 30 shows that although the PIP errors for the Taylor series method are higher – the resultant acceleration and divert requirements are lower for the Taylor series approach. Again the reason for this anomaly is due to the fact that the apparent target maneuver due to the boosting ICBM is more important than the PIP error in this example.

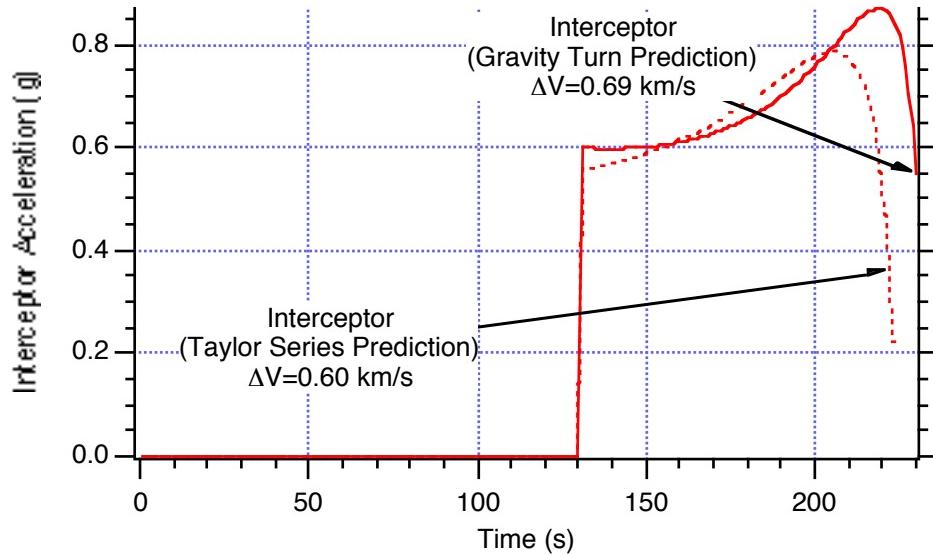


Figure 30 Acceleration and divert are slightly smaller using Taylor series PIP algorithm even though PIP errors are much larger

## Summary

This paper compares the gravity turn and Taylor series method of predicting where a boosting target will be in the future. It was shown that the superiority of one method over the other was dependent on how many stages the threat had and whether Lambert or GEM guidance was used by the threat. It was shown that the gravity turn method of prediction causes the interceptor to lead the target by a larger amount than if the Taylor series method of prediction was used. The longer lead lengthens the time to intercept which can lead to larger divert requirements of a pursuing interceptor because of the apparent target maneuver of the boosting target. Under these circumstances it is possible that a prediction method which yields smaller PIP errors may in fact yield larger interceptor divert requirements.

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